Homework 1

Due: Wednesday, September 16

Each problem is worth 10 points. To get the full credit, write complete, detailed solutions. You may use any of the results from the class without a proof, but you have to state them explicitly.

Problem 1. Let $f : \mathbb{R} \to \mathbb{R}$ be the map $f : x \mapsto (x+1)^2$. Compute the inverse image sets $f^{-1}(A)$ of the following sets A:

a) $\{-9\}$,

b) $\{-1, 0, 4\}$,

c) $[0, +\infty) = \{x \in \mathbb{R} : \ x \geqslant 0\}$.

Problem 2. Let $f : X \to Y$ be a map, and consider $A_1, A_2 \subset X$.

a) Prove that $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$.

b) Prove that $f(A_1 \cap A_2) \subset f(A_1) \cap f(A_2)$ and give an example in which equality fails.

Problem 3. Suppose that $S = \{1, 2, 3\}$. Define a map $\pi : S \to S$ by

$$\pi(1) = 2,$$
$$\pi(2) = 3,$$
$$\pi(3) = 1.$$

a) Find the images of elements of S under the maps $(\pi \circ \pi)$ and $(\pi \circ \pi \circ \pi)$.

b) Is π surjective? injective?. If it is bijective, find π^{-1}.

Problem 4. Let $f : X \to Y$ and $g : Y \to Z$ be maps between sets. Prove that

a) If $g \circ f$ is injective, then f is injective;

b) If $g \circ f$ is surjective then g is surjective.

Problem 5. Let $f : A \to B$ and $g : B \to C$ be invertible mappings; that is, mappings such that f^{-1} and g^{-1} exist. Show that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.