Homework 10

Due: Wednesday, November 25

Each problem is worth 10 points. To get the full credit, write complete, detailed solutions. You may use any of the results from the class without a proof, but you have to state them explicitly.

Problem 1. Let $H \subset D_n$ be the subgroup of the dihedral group, consisting of all rotations:

$$H = \{r^0, r^1, \ldots, r^{n-1}\}.$$

Prove that H is a *normal subgroup*, and describe its cosets.

Problem 2. Enumerate the vertices of a square $\{1, 2, 3, 4\}$ and consider the dihedral group D_4 as a subgroup of the permutation group S_4. Figure out whether D_4 is a normal subgroup or not.

Problem 3. A group G is called *simple*, if its only normal subgroups are $\{e\}$ and G. Give (with a proof) an example of two different simple abelian groups.

Problem 4. Using the fact that $A_3 \subset S_3$ is the only proper normal subgroup in S_3, prove that there is no surjective homomorphism $f\colon S_3 \to (\mathbb{Z}_3, +)$.

Hint: think of what $\mathrm{Ker}(f)$ could be.

Problem 5. Prove that $H = \{[0]_{15}, [5]_{15}, [10]_{15}\} \subset \mathbb{Z}_{15}$ is a normal subgroup. Construct a homomorphism $f\colon \mathbb{Z}_{15} \to \mathbb{Z}_5$ with $\mathrm{Ker}(f) = H$.