Homework 11

Due: Wednesday, December 2

Each problem is worth 10 points. To get the full credit, write complete, detailed solutions. You may use any of the results from the class without a proof, but you have to state them explicitly.

Problem 1. Let D_n be a dihedral group and $N \subset D_n$ the subgroup consisting of all rotations. Prove that N is *normal*.

Problem 2. Let $N \subset D_n$ be as in problem 1. Prove that the quotient group (which is well-defined since $N \subset D_n$ is normal) is isomorphic to $\mathbb{Z}_2$.

Problem 3. Consider a multiplicative group $(\mathbb{R}\backslash\{0\}, \cdot)$. Define a homomorphism

$$f\colon \mathbb{R}\backslash\{0\} \to \mathbb{R}\backslash\{0\} \qquad f(x) = x^2.$$

Find $\mathrm{Ker}(f)$ and $\mathrm{Im}(f)$ and use the first isomorphism theorem to identify

$$(\mathbb{R}\backslash\{0\})/\mathrm{Ker}(f).$$

Problem 4. Let $f\colon G \to G'$ be a nontrivial[1] homomorphism. Given that $|G| = 15$ and $|G'| = 18$, find the size of $\mathrm{Ker}(f)$.

Problem 5. Let G be a group and H a normal subgroup of G such that H and G/H are abelian. Is it true that G is abelian?

[1] The *trivial* homomorphism is the one sending all the elements in G to the neutral element in G'