Homework 2

Due: Wednesday, September 23

Each problem is worth 10 points. To get the full credit, write complete, detailed solutions. You may use any of the results from the class without a proof, but you have to state them explicitly.

Problem 1. *a*) Given any $a \in \mathbb{Z}_8$ what are possible values of $a^2 \in \mathbb{Z}_8$?

b) Prove that equation $x^2 - 8y^2 = 5$ does not have solutions in $\mathbb{Z}$.

Problem 2. Using Euclidean algorithm find $\gcd(36, 235)$ and provide a solution in $\mathbb{Z}$ to

$$36u + 235v = \gcd(36, 235).$$

Problem 3. Solve in $\mathbb{Z}_{235}$ equation

$$36 \cdot x \equiv 7 \mod 235.$$

Problem 4. *a*) Solve equation $x^2 = -1$ in $\mathbb{Z}_{13}$.

b) Let $n \in \mathbb{Z}$ be a positive integer. Prove that equation

$$xy = 0$$

for $x, y \in \mathbb{Z}_n$ has nonzero solutions if and only if n is **not** prime.

Problem 5. Define the least common multiple of two positive integers $a, b \in \mathbb{Z}$ to be the smallest positive integer m such that a and b divide m. Denote this integer by $\operatorname{lcm}(a, b)$. Prove that

$$\gcd(a, b)\operatorname{lcm}(a, b) = ab.$$

Hint: express $\gcd(a, b)$ and $\operatorname{lcm}(a, b)$ in terms of the prime factorizations of a and b.