Homework 3

Due: Wednesday, September 30

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Problem 1. *a*) Describe the set $\mathbb{Z}_{14}^{\times}$. Give an inverse for each of its elements.

b) The same question for $\mathbb{Z}_{17}^{\times}$.

Problem 2. We define a law of composition on $\mathbb{R}$ by

$$x * y := x^2 + y^2.$$

Is it associative? commutative? Does it admit an identity?

Problem 3. Let $(G, *)$ be a group. Prove that for any $x_1, \ldots, x_k \in G$

$$(x_1 * x_2 * \cdots * x_k)^{-1} = x_k^{-1} * \cdots * x_2^{-1} * x_1^{-1}.$$

Problem 4. Find an example of a group $(G, *)$ and elements $x, y \in G$ such that

$$(x * y)^2 \neq (x^2) * (y^2)$$

in G.

Problem 5. List all the symmetries (rigid motions) of a non-square rectangle. Prove that they form a commutative group under composition.