Homework 6

Due: Wednesday, October 28

Each problem is worth 10 points. To get the full credit, write complete, detailed solutions. You may use any of the results from the class without a proof, but you have to state them explicitly.

Problem 1. Prove that for coprime $n, m \in \mathbb{Z}$ group[1] $\mathbb{Z}_n \times \mathbb{Z}_m$ is isomorphic to the group $\mathbb{Z}_{nm}$.

Problem 2. Prove that $(\mathbb{Z}_8^\times, \times)$ is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$.

Problem 3. Let $G = GL_2(\mathbb{R})$ be a group of 2×2 matrices, and

$$H = SL_2(\mathbb{R}) := \{A \in GL_2(\mathbb{R}) \mid \det(A) = 1\}$$

be a subgroup of matrices with determinant 1. Describe the partition of G into the left cosets of H. Does the answer change if we consider right cosets instead?

Problem 4. G is a finite group with elements x and y of orders 5 and 7 respectively. Prove that $|G| \geqslant 35$.

Problem 5. Let $(\mathbb{Z}_{60}^\times, \times)$ be the multiplicative group of units in $\mathbb{Z}_{60}$. Prove that for any $a \in \mathbb{Z}_{60}^\times$ we have

$$a^{16} = 1 \mod 60.$$

Hint: check that $|\mathbb{Z}_{60}^\times| = 16$ and use Lagrange's theorem

[1] Here as usual we consider groups $\mathbb{Z}_k$ with respect to addition