Homework 7

Due: Wednesday, November 4

Each problem is worth 10 points. To get the full credit, write complete, detailed solutions. You may use any of the results from the class without a proof, but you have to state them explicitly.

Problem 1. For each of the following permutations find their decomposition into independent cycles, and compute σ^{100} and σ^{-1}:

$$a)\ \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 6 & 2 & 1 & 3 & 5 \end{pmatrix},\ b)\ \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 6 & 4 \end{pmatrix},\ c)\ \sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 2 & 1 & 3 \end{pmatrix}.$$

Problem 2. Let $(a_1 a_2 \dots a_k) \in S_n$ be any cycle of length k. Prove that in S_n we have the following identity:

$$(a_1 a_2 \dots a_k) = (a_1 a_k)(a_1 a_{k-1}) \dots (a_1 a_3)(a_1 a_2)$$

NB. Composition of permutations is applied from right to left.

Problem 3. Prove that any permutation $\sigma \in S_n$ can be factored as a product of not necessarily independent cycles of length 2 (cycles of length 2 are called *transpositions*).

Problem 4. Figure out (with a proof) whether the following statement is true of false.

If $\sigma \in S_n$ is a cycle, then σ^2 is also a cycle.

Problem 5. From the class we know that possible orders of elements in the group S_4 are $1, 2, 3, 4$. Find the number of elements of order d for every $d \in \{1, 2, 3, 4\}$.

Hint: your counts should add up to $|S_4| = 4! = 24$