Homework 8

Due: Wednesday, November 11

Each problem is worth 10 points. To get the full credit, write complete, detailed solutions. You may use any of the results from the class without a proof, but you have to state them explicitly.

Problem 1. Find all possible orders of elements in the alternating group A_5 (group of even permutations of five elements).

Problem 2. Let $(1,2)$ be a transposition, and $\sigma \in S_n$ any permutation. Prove that the product

$$\sigma(1,2)\sigma^{-1}$$

is the transposition swapping $\sigma(1)$ and $\sigma(2)$.

Problem 3. Let $H \subset S_n$ be a *normal* subgroup containing transposition $(1,2)$. Prove that H must coincide with the whole S_n.

Hint: use the previous problem and the fact that H is normal to show that H must contain *all* transpositions

Problem 4. Twelve volumes of Encyclopedia Britannica are on the shelve in the order $12,1,2,3,\ldots,11$. George is allowed to swap adjacent pairs of volumes, i.e., change $\ldots,a,b,c,d,\ldots$ to $\ldots,c,d,a,b,\ldots$.

a) Prove that George will be able to obtain only the permutations of $\{1,2,3,\ldots,12\}$ that are of the same parity as the initial permutation.

b) Prove that George will never be able to put all the volumes in the correct order $1,2,3,\ldots,12$.

Problem 5. Prove that

$$N = \{\mathrm{id},(12)(34),(13)(24),(14)(23)\} \subset S_4$$

is a subgroup.

*Optional question: show that N is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$.