Homework 9

Due: Wednesday, November 18

Each problem is worth 10 points. To get the full credit, write complete, detailed solutions. You may use any of the results from the class without a proof, but you have to state them explicitly.

Problem 1. In a dihedral group D_4 simplify r^2sr^3s

Hint: use the identity $(sr)^2 = \mathrm{id}$

Problem 2. Let G be any group. For g_1 , $g_2 \in G$ define $g_1 \sim g_2$ if there exists $h \in G$ such that $g_1 = hg_2h^{-1}$. In this case we say that g_1 is *conjugate* to g_2.

Prove that $\sim$ is an equivalence relation on G.

Problem 3. Prove that two permutations σ_1 and σ_2 are conjugate to each other if and only if they have the same lengths of cycles in their independent cycles factorizations.

For examples, permutations

$$(123)(45)(67) \quad \text{and} \quad (12)(345)(67)$$

both have lengths $3, 2, 2$ and thus are conjugate to each other.

Problem 4. Prove that subgroup $N \subset S_4$

$$N = \{\mathrm{id}, (12)(34), (13)(24), (14)(23)\} \subset S_4$$

is normal.

Hint: use the previous problem

Problem 5. Let G be a group and fix an element $g \in G$ Prove that the map

$$i_g\colon G \to G \quad i_g(x) := gxg^{-1}$$

is an isomorphism (such isomorphisms are called *inner automorphisms*).