Homework 10

Due: Tuesday, November 26

Each problem is worth 10 points. To get the full credit, write complete, detailed solutions. You may use any of the results from the class without a proof, but you have to state them explicitly.

Problem 1. Prove that $\mathbb{D}$ and $\mathbb{C}$ are not conformally equivalent.

Problem 2. Does there exists a holomorphic surjection $\mathbb{D} \to \mathbb{C}$?

Problem 3. Prove that

$$f(z) := -\frac{1}{2}\left(z + \frac{1}{z}\right)$$

is a conformal map from the half disk $\{z \mid |z| < 1, \operatorname{Im}(z) > 0\}$ to the upper half-plane $\mathbb{H}$.

Problem 4. A holomorphic function $f : U \to V$ is *local bijection* if for every $z \in U$ there exists an open disk $D \subset U$ centered at z_0 such that $f : D \to f(D)$ is a bijection.

Prove that $f : U \to V$ is a local bijection if and only if $f'(z) \neq 0$ for $z \in U$.

Hint: use Rouche's theorem.

Problem 5. Prove that family of functions $\mathcal{F} = \{z^n\}_{n\in\mathbb{N}}$ is normal in the unit disk $\mathbb{D}$ but not normal in any larger open set.