# Homework 11

**Due:** Tuesday, December 3

To get the full credit, write complete, detailed solutions. You may use any of the results from the class without a proof, but you have to state them explicitly.

**Problem 1** (20 points)**.** Let $f(z) = \log(z)$ be the principle branch of logarithm in $U = \mathbb{C} - (-\infty; 0]$.

*a*) Find two Taylor's series for $f(z)$ centered at $z_0 = -1 - \boldsymbol{i}$ and $z_0 = -1 + \boldsymbol{i}$.

*b*) What is their radius of convergence?

*c*) What is their value at $z = -1$?

**Problem 2** (10 points)**.** Let $[f]_1$ be a germ of the function $f(z) = \sqrt{z} + \sqrt[3]{z}$ at $z = 1$. How many germs one can obtain at $z = -1$ by analytically continuing $[f]_1$ along all possible paths in $\mathbb{C} - \{0\}$.

**Problem* 3** (Optional, no credit)**.** Fix $\alpha > 0$. Prove that function

$$f(z) = \sum 2^{-n\alpha} z^{2^n}$$

is holomorphic in $B_1(0)$, extends to a continuous function on $\overline{B}_1(0)$, yet it does not extend to a holomorphic function in any larger open set.

Hint: Restriction of $f(z)$ to the unit circle is continuous but nowhere differentiable.